

Scaling properties of the irreversible magnetization curves of high-temperature superconductors

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A one-parameter scaling of the magnetization curves of the high-temperature superconductor Tl-Ba-Ca-Cu-O causes all data points in the irreversible regime to collapse into a single curve  $M = H^* f_{\pm}(H/H^*)$ . The scaling field  $H^*$  is inversely proportional to temperature and  $f_+$  and  $f_-$  are the scaling functions for fields  $H$  above and below  $H^*$ , respectively. The results are explained in terms of the Bean model. We argue that these features are common to all type-II superconductors.

The magnetization in the Abrikosov mixed state in ideal type-II superconductors (SC) is reversible. In reality, however, most type-II SC are characterized by irreversible magnetic features which result from flux trapping at pinning centers.<sup>1</sup> Typical pinning energies in conventional type-II SC are of order 1 eV. In contrast, the new high-temperature superconductors (HTSC) are characterized by very low pinning energies<sup>2,3</sup> of order 0.01 eV. Perhaps the most surprising demonstration of the weak pinning forces is the existence of a wide reversible regime in the field-temperature ( $H$ - $T$ ) phase diagram<sup>2,4</sup> of HTSC. The "irreversibility line," the line which separates the reversible from the irreversible regimes, is determined experimentally by measuring the temperature  $T_{irr}(H)$  above which the zero-field-cooled (ZFC) and the field-cooled (FC) susceptibilities coincide.

The magnetization in the reversible state is described by the Abrikosov equations, whereas the field and temperature dependence of the magnetization in the irreversible regime is conventionally described by the Bean model.<sup>5</sup> The main purpose of this work is to explore experimentally the validity of the Bean model in HTSC. We present here a detailed study of the field dependence of the magnetization of a ceramic Tl<sub>2</sub>Ba<sub>2</sub>Ca<sub>2</sub>Cu<sub>3</sub>O<sub>10</sub> with a superconducting transition temperature  $T_c = 113$  K. We find simple "scaling" features of the magnetic isotherms in the irreversible regime and show that, under certain conditions, these features are intrinsic to the Bean model and are thus expected to occur for all type-II superconductors.

In Ref. 6 we describe sample preparation and details of magnetic measurements below and above  $T_c$ . The magnetization  $M$  of the 285-mg disklike sample is measured with a commercial SHE superconducting quantum interference device magnetometer with the field oriented in the disk plane in order to minimize demagnetization corrections. We follow a conventional procedure: The sample is first cooled in zero field to low temperature, a field  $H$  ( $30 \text{ Oe} \leq H \leq 40 \text{ kOe}$ ) is then applied and the ZFC magnetization is recorded up to  $T = 2T_c$ . The sample is then cooled in the presence of the same field and the FC branch of the magnetization is recorded while the sample is warming up to  $T_c$ . The magnetization curves

$M(H)$  are determined from the recorded  $M(T)$  values at a constant temperature. By a direct measurement of  $M$  as a function of field at a constant temperature (70 K), we have verified<sup>6</sup> that the procedure described above indeed yields reliable values for  $M(H)$ .

Figure 1 exhibits typical  $M$  vs  $H$  data at temperatures ranging from 30 to 80 K. Corrections for demagnetization fields have a negligible effect (less than 2%) on data points. We note that even at the lowest temperature there is no linear regime in the magnetization, indicating that flux is penetrating into the sample even in the low-field low-temperature limit. This observation is consistent with the recent estimate of  $H_{c1} \leq 5$  Oe at low temperatures in ceramic samples and in single crystals of similar composition.<sup>7</sup> We also note that as temperature decreases, the minimum in the magnetization curves is pushed to higher fields and to larger absolute values. This last observation is the basis for our scaling procedure. We define  $H_m$  as the field for which  $M$  reaches its minimum value  $M_m$ . We then scale the field values of each isotherm by  $H_m$  and, similarly, we scale the magnetization values by  $M_m$ . As a

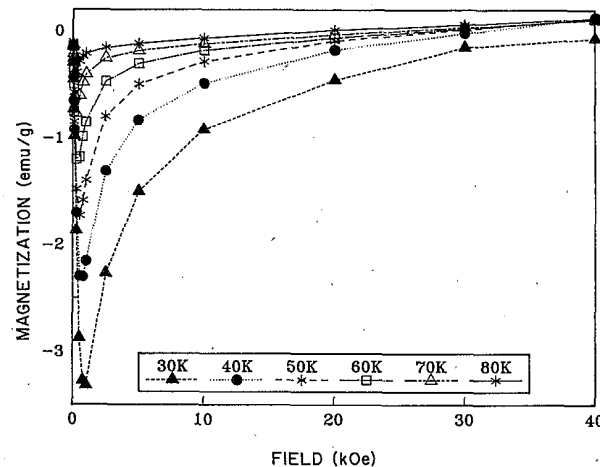


FIG. 1. Magnetization curves for Tl-Ba-Ca-Cu-O at the indicated temperatures.

result of this simple scaling procedure, data points for a wide range of temperatures and fields collapse into a single curve, as shown in Fig. 2. The success of this scaling procedure is even more impressive once we appreciate the implication of the temperature dependence of the two scaling parameters. The inset to Fig. 2 demonstrates that both  $H_m$  and  $M_m$  scale with the inverse of the temperature. In other words,  $M_m \propto H_m$ , and hence the scaling procedure is actually a one-parameter scaling.

Deviations from scaling are observed in the high-temperature high-field limit. This is demonstrated in Fig. 3 where we show the effect of the scaling procedure on all the data points of Fig. 1. The main observation here is that deviations occur at higher fields as temperature decreases. The temperature dependence of the fields for which we first observe deviations from scaling are denoted by triangles in the field-temperature ( $H$ - $T$ ) phase diagram of Fig. 4. This figure also describes the irreversibility line (squares) deduced from the observed deviation (the

“branching” point) of the ZFC and FC curves. (We have used a similar criterion for both experiments, namely a 0.1% deviation.) The apparent similarity in the  $H$ - $T$  dependence of the crosses and the squares in Fig. 4 demonstrates that the scaling features of Fig. 2 are related to the irreversible regime.

The scaling features of the irreversible magnetization can be explained in the framework of the Bean model. In this model, pinning centers prevent homogeneous distribution of fluxons inside the sample, which results in a linear drop of the local magnetic field  $h$  with the distance  $x$  from the surface until it reaches the value  $H_{c1}$ . In its simple version, the model assumes that the critical current  $J_c$  is field independent leading to a linear dependence of  $h$  on  $x$ . A more realistic model<sup>3</sup> takes  $J_c = J_{c1} h^{-n}$ , where  $J_{c1}$  is the maximum critical current at a given temperature and  $n$  is a phenomenological power, typically 0.5-1 in experiments. Such a recent extension of the Bean model yields<sup>3</sup> for a slab of thickness  $D$ ,

$$H + 4\pi M = \frac{2}{CD} \frac{n+1}{n+2} (H^{n+2} - H_{c1}^{n+2}), \quad H_{c1} \leq H \leq H^*, \quad (1)$$

$$H + 4\pi M = \frac{2}{CD} \frac{n+1}{n+2} \left[ H^{n+2} - \left( H^{n+1} - \frac{CD}{2} \right)^{(n+2)/(n+1)} \right], \quad H \geq H^*, \quad (2)$$

where

$$C \equiv (4\pi/10)(n+1)J_{c1}H_{c1}^n$$

and

$$H^* \equiv (CD/2 + H_{c1}^{n+1})^{1/(n+1)}$$

is the lowest field for which currents flow through the entire volume of the sample. Note that for  $n=0$ , the original Bean equations are recovered. We choose the scaling

field to be  $(CD/2)^{1/(n+1)} \approx H^*$ . The scaling field is thus the lowest field for which flux penetrates into the entire volume of the sample. Note that in the Bean model  $H^*$  is proportional to, but larger than,  $H_m$ . Also note that in the definition of the scaling field, we assume that  $H_{c1}$  is small compared to the experimental fields. As discussed above, this is indeed the case here, even in the low-field limit of our experiment. By scaling both sides of Eqs. (1) and (2) by the scaling field  $H^*$ , we find

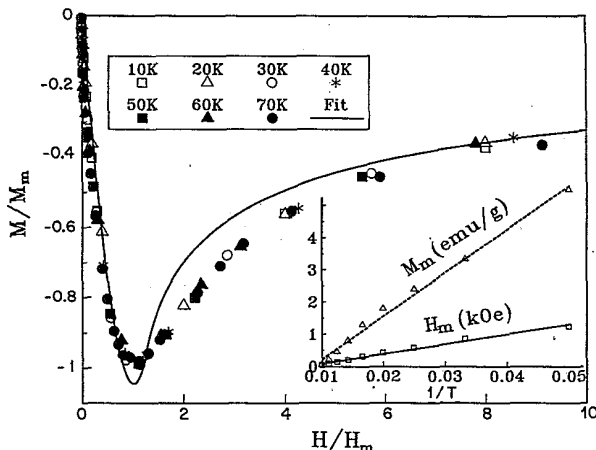


FIG. 2. Scaled magnetization curves for Tl-Ba-Ca-Cu-O ( $H/H_m \leq 10$ ). Solid line is a fit to Eqs. (3) and (4) with  $n=0.45$ . Inset: The scaling parameters  $H_m$  and  $M_m$  as a function of the inverse temperature.

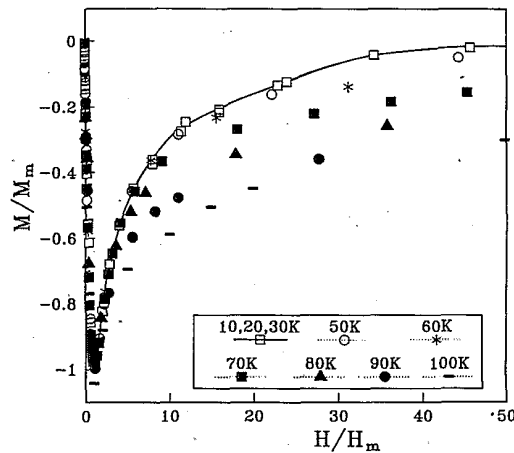


FIG. 3. Scaled magnetization curves for Tl-Ba-Ca-Cu-O ( $H/H_m \leq 60$ ). The solid line connects data points of the lowest temperatures. Deviations from scaling are observed in the high-temperature high-field limit.

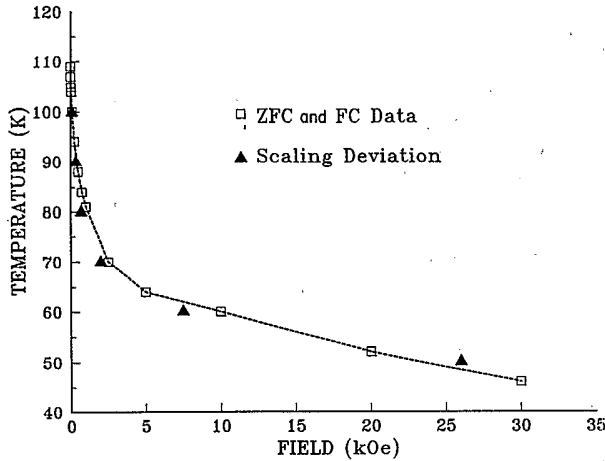


FIG. 4. Dashed line: Irreversibility line for Tl-Ba-Ca-Cu-O deduced from the branching point of the ZFC and FC curves at a constant field. Triangles: Deviations from scaling in Fig. 3.

$$\frac{4\pi M}{H^*} = - \left( \frac{H}{H^*} \right) + \frac{n+1}{n+2} \left( \frac{H}{H^*} \right)^{n+2}, \quad (3)$$

$$\frac{4\pi M}{H^*} = - \left( \frac{H}{H^*} \right) + \frac{n+1}{n+2} \left\{ \left( \frac{H}{H^*} \right)^{n+2} - \left[ \left( \frac{H}{H^*} \right)^{n+1} - 1 \right]^{(n+2)/(n+1)} \right\}. \quad (4)$$

In summary,

$$4\pi M = H^* f_{\pm}(H/H^*), \quad (5)$$

where  $f_-$  and  $f_+$  are the scaling functions for  $H/H^* \leq 1$  and for  $H/H^* \geq 1$ , respectively. Thus, a one-parameter scaling is an intrinsic feature of the extended Bean model, provided that  $H_{c1}$  in Eqs. (1) and (2) might be neglected.

We fit the scaled data of Fig. 2 to Eqs. (3) and (4) and find that the data are best described by an exponent  $n=0.45$  (solid line in Fig. 2), which is in a reasonable agreement with the weak field dependence of  $J_c$  found in conventional<sup>1</sup> as well as in high-temperature superconductors.<sup>3,8</sup>

An interesting implication of the above discussion is the possibility of determining the temperature dependence of the critical current from the temperature dependence of the scaling field ( $H^* \propto T^{-1}$ ; see the inset of Fig. 2). Using the expressions for  $H^*$  and for  $C$  we find  $H^* \propto C^{1/(n+1)} \propto J_c^{1/(n+1)}$ . Thus, it is obvious that  $J_c \propto T^{-(n+1)}$ . This prediction provides a possibility for an independent determination of the exponent  $n$ . Direct measurements of

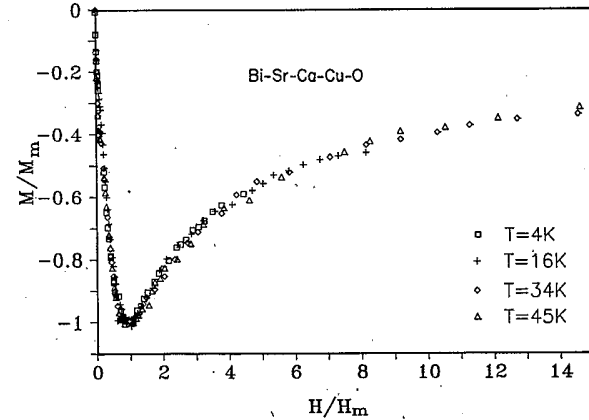


FIG. 5. Scaled magnetization curves for Bi-Sr-Ca-Cu-O ( $H/H_m \leq 10$ ) for the indicated isotherms.

$J_c(T)$  are, of course, essential to verify this prediction.

In conclusion, we have demonstrated a one-parameter scaling of the magnetization curves of Tl-Ba-Ca-Cu-O in the irreversible regime of the mixed phase. The main qualitative observation in the experimental work is that  $M(H, T)$  depends on temperature  $T$  only through a single scaling field variable  $H^*(T)$ . We show that this scaling property is built into the Bean model. We are now studying the plausible possibility that the scaling features are a more general property, independent of the particular model. This implies that similar behavior should be observed in other type-II superconductors. Preliminary results, shown in Fig. 5, for a ceramic Bi-Sr-Ca-Cu-O sample with  $T_c = 114$  K indeed show similar scaling features. Other samples are currently being investigated.

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