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Hall probe response to a distribution of vortices in superconductor*

A.Reiderman, Y.Wolfus, Y.Abulafia and Y.Yeshurun,

Department of Physics, Bar-Ilan Univ., 52900, Ramat-Gan, Israel

Based on an analytical approach, an approximation of the Hall probe local sensitivity function by the square uniformly sensitive region w×w, where w is the distance between the Hall electrodes, is given. A simple formula for the Hall probe response to a distribution of vortices in superconductors is proposed.

The Hall-probe technique has shown its efficiency as a tool for mapping out the magnetic field distributions of the millimeter size high-temperature superconductor crystals and thin films [1-3]. A spatial inhomogeneity period of local field in the measurements is usually comparable to the probe size. Therefore a local sensitivity consideration is required for the quantitative characterization of the field source in this case. We present a simple approximate technique for this consideration.

The problem of the inhomogeneous regime of the Hall probe operation is described by the following set of equations[4]

$$\rho \mathbf{j}=\mathbf{E}+R_h(\mathbf{j}\times\mathbf{B}) \,, \qquad \nabla\times\mathbf{E}=0 \,, \quad \nabla\mathbf{j}=0 \,, \tag{1}$$

with the boundary conditions

$$j_y=0 \ \text{ for } y=\pm w/2$$
$$\text{and } E_y=0 \ \text{ for } x=\pm c/2 \,,$$

where $\mathbf{j}$ is the Hall probe current density; ρ is the resistivity; R_h is the Hall probe coefficient; $\mathbf{E}$ is the electric field intensity which determines the Hall voltage; $\mathbf{B}$ is the measured local field; x and y are the coordinates in the probe plane , x is directed along the driving current; c and w are the probe dimensions in X-axes and Y-axes directions respectively. (Note: the origin of the coordinates is chosen to be the center point of the line between the Hall electrodes).

We assume that the field $\mathbf{B}$ is small enough to produce only small perturbation in the Hall probe current. Introducing then the potential function $\phi' = \phi - R_h j_o \int_{-w/2}^{y} B(x,y)d\eta$, where ϕ is determine by the equation $\nabla\phi = -\mathbf{E}$, we obtain the following analytical solution for the Hall voltage

$$V_h = R_h j_o \int_{-w/2}^{w/2} B(x=0,y)dy - 2\int_{-w/2}^{w/2}dy' \int_{-c/2}^{c/2}dx'\, G_b \int_{-w/2}^{y'} d\eta\, \partial^2 B(x',\eta)/\partial x'^2, \tag{2}$$

with G_b= G(x=0, y=w, x', y'-w/2), where G is the Green's function for the Laplace equation in rectangle w×c [5], j_o is the current density for B=0.

When mapping the magnetic field of a superconductor, the question is raised of how sensitive is the Hall probe to the vortex field. Let $\mathbf{B}$ be the magnetic field of an isolated

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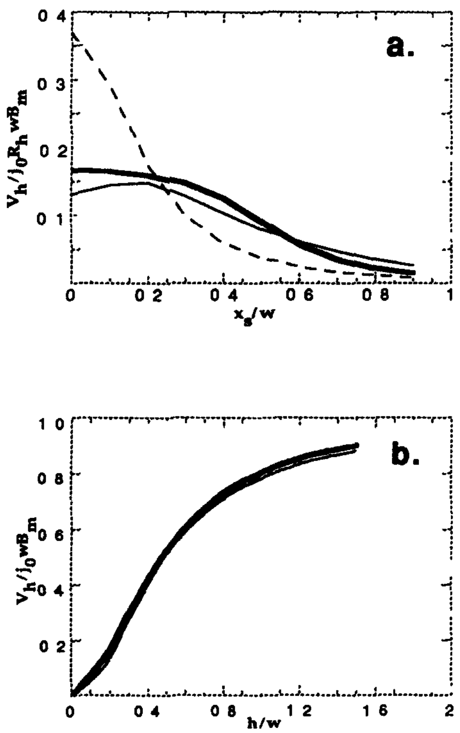

Figure1. Normalized Hall probe voltage versus vortex position x_s at $y_s=0$, $h=0$ (a) and probe-to-sample separation h at $x_s=0$, $y_s=0$ (b). Normal solid line is the result of calculation according to (2); bold line is the average of B according to (3) with N=1. Dimensions ratio $c/w >> 1$

vortex. With a certain approximation this field can be treated as the magnetic pole field [3] with z-component $B_v = (\Phi_0 h/2\pi)/|r-r_s|^3$, where r_s and r determine the pole position and field point respectively, h is the probe-to-sample surface separation. Substituting B_v in the expression for V_h we obtain the Hall probe response to the vortex field. The fig.1 presents the dependence of normalized Hall voltage: $V_h/j_0R_hwB_m$ (B_m is the maximum of B_v) on the vortex position. The results show that the Hall probe can be approximately treated as uniformly sensitive in a square region of w×w and not relatively narrow region near the line between Hall electrodes as it can be expected without taking into account an inhomogeneous perturbation in probe current (broken line in the fig.1). The result means also that the dimension ξ of the Hall electrodes have little effect on the registered Hall voltage and spatial resolution of the probe if $|\xi| \le w/2$. Note that the sensitivity in the square w×w behaves as the average of B(x,y) over this region (bold line).

Based on this local sensitivity behavior we can give an approximate formula for the Hall probe response to an assembly of N arbitrary positioned (at the points $\mathbf{r_i}$) vortices

$$V_h = (R_h j_0/w) \sum_{i=1}^{N} \int_{S_s} B_v(\mathbf{r}-\mathbf{r_i})ds\,, \qquad (3)$$

where S_s is the square $|x| \le w/2$; $|y| \le w/2$.

For $h << w$ we have $V_h = N_s \Phi_0 R_h j_0/w$, where N_s is the number of vortices in S_s.

In conclusion we have shown a square uniformly sensitive region to be a good approximation for local sensitivity of the Hall probe. Hence a simple formula can be given for the quantitative characterization of a distribution of vortices.

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