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Fractal excitations in dilute magnets

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ABSTRACT

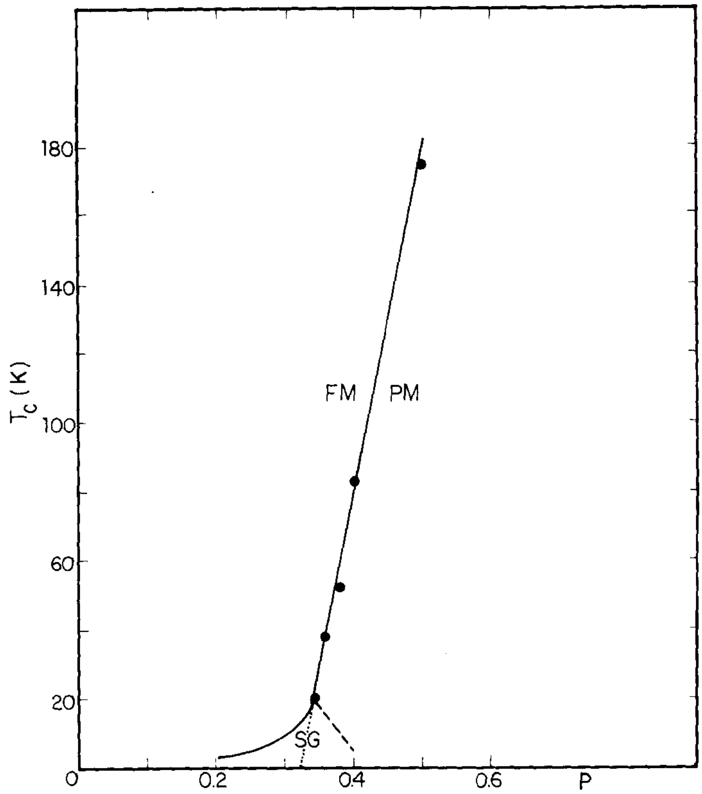
The low-temperature magnetization in a series of amorphous $(Co_pNi_{1-p})_{75}P_{16}B_6Al_3$ alloys ($0{\cdot}34 \leqslant p \leqslant 0{\cdot}5$) deviates strongly from the Bloch $T^{3/2}$ law. These deviations are shown to result from short-wavelength excitations ('fractons') for which the random magnetic network looks fractal.

Decades ago it was pointed out that well-defined spin waves are expected to propagate in disordered magnetic systems provided that their wavelength is long compared with the length scale of the disorder (Kittel 1959). Only recently has the cross-over between long-wavelength and short-wavelength excitations been treated (Alexander, Bernasconi, Schneider and Orbach 1981, Alexander, Laermans, Orbach and Rosenberg 1983, Aharony, Alexander, Entin-Wohlman and Orbach 1985). It has been suggested that for short-wavelength excitations the random magnetic network looks fractal (Orbach 1986). This change in the dimensionality of the system implies a cross-over in the density of states (DOS) for the excitations from a spin-wave DOS in the long-wavelength regime to the so-called 'fracton' DOS for the short-wavelength excitations. It is the purpose of this paper to demonstrate experimentally the cross-over in the density of states of the magnetic excitations (Salamon and Yeshurun 1987).

The magnetic system under investigation is amorphous $(Co_pNi_{1-p})_{75}P_{16}B_6Al_3$ (Yeshurun, Rao, Salamon and Chen 1981). In this system nickel is non-magnetic, and it serves to dilute the magnetic cobalt atoms. The dilution creates a percolating magnetic structure with the percolation threshold for ferromagnetism at $p_c \simeq 0{\cdot}325$. The main advantage in studying *amorphous* ferromagnets lies in the fact that p can be changed continuously, thus enabling a study of the magnetic excitation much closer to p_c than is achieved using other approaches (Uemura and Birgeneau 1986).

In fig. 1 we show the magnetic phase diagram in zero field for $(Co_pNi_{1-p})_{75}P_{16}B_6Al_3$. The circles in the figure denote the ferromagnetic transitions for the samples under investigation in this work. It should be noted that magnetic order (spin glass) is found for samples below p_c. This indicates that, owing to the relative long-range RKKY interactions which dominate the magnetic behaviour for $p < p_c$, the spin-glass state has its own percolation threshold. A spin-glass phase also appears at low temperatures for the samples under investigation ($p \gtrsim p_c$). These are samples in the well-known 're-entrant' ferromagnetic phase. To avoid complications arising from the spin-glass state, we analyse high-field data only. The field ($H = 10$ kOe) is well above

Fig. 1



The zero-field magnetic phase diagram for amorphous $(Co_pNi_{1-p})_{75}P_{16}B_6Al_3$, based mainly on the results of Yeshurun *et al.* (1981). The full circles denote the ferromagnetic (FM) transitions for the sample under investigation. The dotted line is an extrapolation to the FM percolation threshold. The broken line is the re-entry line, which is 'destroyed' by the field, $H = 10$ kOe.

the de Almeida–Thouless line for this sample, ensuring that the spin-glass phase is destroyed (Sompolinsky 1981).

The experimental approach in this work is based on the effect of magnetic excitations on low-temperature magnetization. Since each spin excitation reduces the magnetization of the ferromagnet by one Bohr magneton, the well known Bloch's law,

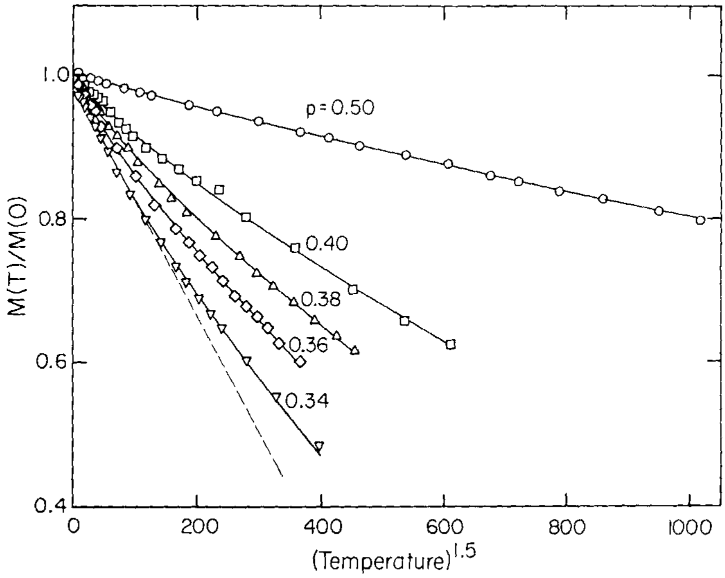
$$M(T)/M(0) = 1 - BT^{3/2}, \tag{1}$$

follows from a Bose–Einstein integration of the spin-wave DOS for a d-dimensional system

$$n_{sw} \propto \omega^{(d/2)-1}. \tag{2}$$

Equation (1) has been confirmed experimentally on many crystalline and amorphous (Axe, Shirane, Mizoguchi and Yamauchi 1977) magnets. However, significant deviations from (1) are observed in dilute amorphous magnets (Bhagat, Spano, Chen and Rao 1980, Yeshurun *et al.* 1981). This is demonstrated in fig. 2 where we plot the

Fig. 2



Reduced magnetization against temperature to the power 3/2. Upward curvature is anomalous. The solid lines are fits to the data using the spin-fracton/magnon density of states. The broken line is the extrapolation of the initial slope for $p=0{\cdot}34$. $H=10$ kOe.

normalized magnetization $M(T)/M(0)$ for amorphous $(\mathrm{Co}_p\mathrm{Ni}_{1-p})_{75}\mathrm{P}_{16}\mathrm{B}_6\mathrm{Al}_3$ $(0{\cdot}34\leqslant p\leqslant 0{\cdot}50)$ as a function of $T^{3/2}$. The dashed line in fig. 2 is the extrapolation of the initial shape of $M(T)/M(0)$ for one of the samples ($p=0{\cdot}34$). This line represents the behaviour expected from (1). The *upward* curvatures which characterize the deviations from the expected straight lines eliminate the possibility that these deviations are due to contributions from hight order $(T^{5/2},\ldots)$ terms; these contributions should cause *downward* curvatures. We thus conclude that deviations from the Bloch law (1) signal changes in the spin waves DOS (2). Following the fractal approach (Alexander *et al.* 1981, 1983, Aharony *et al.* 1985, Orbach 1986), for 'fracton dimensionality' $\bar{\bar{d}}$, the DOS is

$$n_{\mathrm{fr}}\propto \omega^{(\bar{\bar{d}}/2)-1}, \tag{3}$$

where $\bar{\bar{d}}=2d/(2+\theta)$†, and θ is the 'diffusion exponent', i.e. the diffusion constant decreases with distance as $r^{-\theta}$ (Ben-Avraham and Havlin 1982, Gefen, Aharony and Alexander 1983). The cross-over between (2) and (3) occurs at

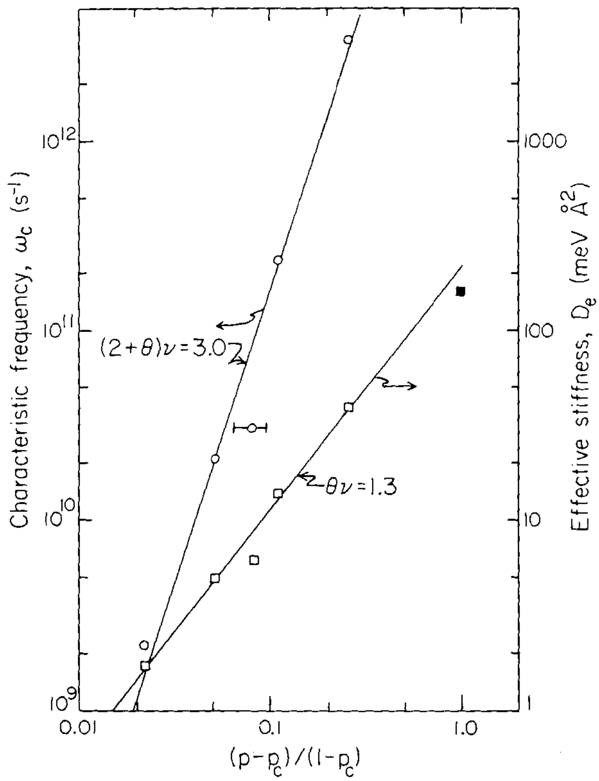
$$\omega_{\mathrm{c}}\propto \xi_{\mathrm{p}}^{-(2+\theta)}, \tag{4}$$

where $\xi_{\mathrm{p}}=\xi_0[(p-p_{\mathrm{c}})/(1-p_{\mathrm{c}})]^{-\nu}$ is the percolation correlation length.

In order to include possible fraction contributions to the temperature dependence of the magnetization, we introduce an effective DOS (Bhagat *et al.* 1980) which

†This expression holds when a contribution from finite clusters is included in the calculations (Alexander *et al.* 1983).

Fig. 3



Effective spin-wave stiffness $D(p)$ and magnon/fracton cross-over frequency $\omega_c(p)$ against reduced concentration. The critical concentration p_c is taken to be 0·325. The solid square is the pure limit value from the normal Bloch law.

interpolates between (2) and (3):

$$n_{\text{eff}} \propto D(p)^{3/2}\omega^{(d/2)-1}(1+\omega/\omega_c)^{(\bar{\bar{d}}-d)/2}, \tag{5}$$

where the stiffness constant

$$D(p) \propto \xi_p^{-\theta}. \tag{6}$$

For the pure limit, $\omega \gg \omega_c$, the spin-wave DOS is recovered. At the other extreme ($\omega \gg \omega_c$), n_{eff} approaches the fraction DOS with a coefficient that is independent of ξ_p. The magnetization is calculated by numerically integrating over the effective density of states:

$$M(T) = M(0) - (g\mu_B)\int \frac{n_{\text{eff}}(\omega)\,d\omega}{\exp\left[(h\omega + g\mu H)/k_B T\right] - 1}. \tag{7}$$

We use (7) to fit the experimental data by treating $D(p)$, $\omega_c(p)$, $\omega_c(p)$ and θ, which appear implicitly in the expression for n_{eff}, as adjustable parameters. Correlations between θ and the other two parameters prevent a simultaneous determination of all three. We fix θ and search for self-consistency in the value of θ deduced independently from (4) and (6). Consistency can be achieved only by fixing the exponent in the range $\theta = 1{\cdot}5 \pm 0{\cdot}1$. The theoretical value of θ ranges from 1·3 at short times (ultra-short wavelengths) to the asymptotic value of 1·7 (Havlin and Ben Avraham 1982). Our experimental value is thus an effective one, owing to averaging of the various contributions.

Values of $D(p)$ and $\omega_c(p)$ obtained from the fits in fig. 2 are plotted against reduced concentration in fig. 3. The data are well represented by $D(p) \propto (p-p_c)^{\theta\nu}$ and $\omega_c(p) \propto (p-p_c)^{(2+\theta)\nu}$ with $\theta\nu = 1{\cdot}3 \pm 0{\cdot}8$ and $(2+\theta)\nu = 3{\cdot}0 \pm 0{\cdot}1$. These values yield $\nu = 0{\cdot}85 \pm 0{\cdot}07$ and $\theta = 1{\cdot}5 \pm 0{\cdot}1$, both of which are consistent with theory for three-dimensional site percolation and with our assumed value of θ.

Recently, Uemura and Birgeneau (1986) used high-resolution neutron scattering from the random antiferromagnet $(Mn_{0\cdot5}Zn_{0\cdot5})F_2$ to demonstrate a cross-over from well defined low-energy spin waves to quasi-localized excitations at high energies. Though this specific sample is relatively far from the percolation threshold, the results of their experiment serve as direct qualitative evidence for the predicted cross-over phenomena. Our experimental results for the diluted amorphous magnets Co_pNi_{1-p} with $p \gtrsim p_c$ are in good *quantitative* agreement with the conjecture of fractons in percolation networks. We thus conclude that the accumulated experimental evidence lends support to the existence of fractons in percolating magnets.

Acknowledgment

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