# Calculated E-I characteristics of HTS pancakes and coils exposed to inhomogeneous magnetic fields

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Abstract. The upper limit of the operating current of LTS solenoids can be estimated as the coordinate of the crossing point of its load line with  $I_C(B)$  line of the superconductor. For HTS coils this approach seems to underestimate the allowable operating current of the coil. A better approach is to obtain a full electric field distribution over the coil and to use it as the base for a more sophisticated coil design criteria. We developed an algorithm and a Matlab program for calculating distributions of the current density, magnetic field and electric field in HTS solenoids made of pancakes, considering the inhomogeneous current density distribution inside the anisotropic tape. I-V curves of several Bi-2223 coils are calculated and good agreeement of the calculated and mesured critical currents,  $I_C$ , and indexes, n, are attained. One can utilize the program in the coil design choosing his own criteria of coil's critical current, e.g., 1) The average electric field  $10^{-4}$  V/m over the coil, 2) The electric field  $10^{-4}$  V/m at the weak point of the coil, 3) The energy dissipation in the entire coil, 4) Distribution of local energy dissipation.

#### 1. Introduction

In recent years HTS Bi-2223 wires characteristics have been improved continuously and their production these days is reaching hundreds of kilometers per year. Fast progress in manufacturing the 2-nd generation HTS (2G) wires has allowed the production of the first long length 2G-wire YBCO coil this year [1]. Thus, problems of HTS coil design and optimization have become relevant. The critical current and averaged E-I curve of the HTS coil are insufficient parameteres of its design. Furthermore, knowing the distributions of current density and electric field in the coil is a necessity for a detailed design of the coil, particularly in the case of cryogen free magnets. Different solutions for this problem were proposed in ref. 2-5.

The upper limit of the operating current of LTS solenoids can be estimated by using the coordinate of the cross point of its load line, related to maximal magnetic field on the windings, with the  $I_c(B)$  line of the superconductor [6]. For HTS coils this approach seems to underestimate the allowable operating current of the coil, because small energy dissipation in the critical point of the HTS winding does not normally results in a quench of the coil. The transition of the HTS coil to normal state occurs when the cooling capacity does not compensate the energy dissipation in the coil or in part of the coil. Therefore, other critical current criteria of the coil have to be regarded. To analyze the behavior of the current carrying HTS coil it is necessary to posses a detailed knowledge of the electric field and

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current density distributions over the entire coil. In this paper we present an algorithm and a Matlab program for calculating distributions of the current density, magnetic field and electric field in HTS solenoids made of pancakes. We consider in the algorithm the inhomogeneous current density distribution inside the anisotropic tape. This inhomogeniouty strongly influences the electric field distribution. A good agreement between the experimental and calculated E-I curves of several coils demonstrates the high efficiency of the program.

#### 2. Results and discussion

#### 2.1. Analysis method

In our method we used numerical integration of the Biot-Savart equation in order to calculate the magnetic field distribution over the entire volume of the coil. The developed algorithm (figure 1) was realized within a Matlab program. Below we describe the algorithm for analyzing one pancake, although it can easily be adapted to a coil with any number of pancakes. The coil consists of  $N_i$  turns. The cross-section of each turn is divided in  $N_e=11$  equal elements along the wide direction of the tape (figure 2). The Matlab program computes in Cartesian coordinates with the origin placed in the center of the coil. When analyzing multipancake coils, the gap between pancakes was introduced in the model.



**Figure 1.** Algorithm of the coil's I-E curve computation

**Figure 3.** Convergence of the computation process

The algorithm includes the following steps:

*Step 1*. Set an initial value of the coil current  $I_{r}$ . Select this value according to the maximal value of magnetic field normal to the tape's plane. This value can be estimated with any FEA software (for example PC-OPERA of Vector Fields Ltd.).

Step 2. Set the first value of the current in the element (l,e) of the tape as  $I_{le} = I_T / N_e$ . The upcoming values  $I_{le}$  will be set as calculated at step 6, after calculating the magnetic field affecting each element (l,e).

*Step 3.* Compute the magnetic field vector  $\mathbf{B}_{l_e}$  for each element (l,e) by numerical integration of the Biot-Savart equation over the volume of the coil.

Step 4. Calculate the component of the vector  $\mathbf{B}_{le}$  -  $\mathbf{B}_{le}^{\perp}$  and the angle  $\boldsymbol{\theta}$  between  $\mathbf{B}_{le}$  and  $\mathbf{B}_{le}^{\perp}$ .

Step 5. By using the database described in section 2.2 calculate parameters  $I_c(l,e)$  and n(l,e), thus attaching an E-I curve to each element (l,e).

Step 6. Compute the currents  $I_{l_e}$  by solving 1 linear and 10 nonlinear equations for each element (l,e).

$$\sum_{e} I_{l,e} = I_T \text{ and } \left( I_{l,1} / I_{l,1(C)} \right)^{n_{l,1}} = \left( I_{l,2} / I_{l,2(C)} \right)^{n_{l,2}} = \dots = \left( I_{l,11} / I_{l,11(C)} \right)^{n_{l,11}} = E_l / E_0$$

Step 7. Calculate the average electric field on the coil as  $\overline{E}_k = \sum_l E_l 2\pi r_l / \sum_l 2\pi r_l$ 

Step 8. Check convergence criterion  $\overline{E}_k - \overline{E}_{k-1} \le \Delta E$ . The convergence process is going very fast when  $\Delta E = 0.00001E$  (figure 3).

Step 9. After finishing the procedure and obtaining the point (E, I) on the E-I curve, go to the subsequent current values to obtain other points (E, I) on the E-I curve and calculate the parameters  $I_c$  and n for the coil.

Actually, two points (E, I) are sufficient for determining an E-I curve according to power law. We have calculated 5 points for each coil and found that all points are well correlated with a power law, as our measurements also show.

In table 1 the computed parameters,  $I_c$  and n, of several coils are compared with those obtained in experiment. Very good agreement between experimental and calculated E-I curves show the high efficiency of this program.

Table 1. Comparison of the calculated and measured parameters of E - I curves

	Coil	Critical current $I_c$ , A		n-value	
		Calculated	Measured	Calculated	Measured
1	Single pancake 10 turns	83.6	84	14.4	13.4
2	Single pancake 20 turns	76.5	75	13	13.5
3	Single pancake 100 turns	66.2	65.4	12.7	13.6
4	Coil 20 turns 2 pancakes	76.4	76	13.1	11.7
5	Coil 300 turns 3 pancakes	46	44	11.5	10.7

#### 2.2. E-I curves of the Bi-2223 tape

E-I curves for all measured values of magnetic flux density and angle  $\theta$  are well described by a power law given by  $E = E_O (I/I_C)^n$ . Thus, we can get the critical current,  $I_c$ , and exponent *n* of an E-I curve for magnetic field in each point (B,  $\theta$ ) using linear approximation. A graph representing the dependency of the critical current,  $I_c$ , on B and  $\theta$  is shown in figure 4. A similar graph can be drawn for exponent *n*.



**Figure 4.** Dependence of the critical current on B and  $\theta$ 

#### 2.3. Distribution of the current density, magnetic and electric field

In this paragraph we demonstrate two examples of the magnetic field, current density and electric field distributions obtained by using the described program for the single pancake (number 3 in table 1) and for three-pancakes coil (number 5). Figure 5 depicts the distribution of the radial and axial components of the magnetic field on the turns of the pancake. The inhomogeneity of the magnetic field results in a strong inhomogeneity of the current density (figure 6). At the same time, a smooth variation of the electric field (figure 7) is observed in the single pancake with a maximal value,  $E_{MAX} = 1.5 \times 10^4 \text{ V/m}$ , that slightly exceeds the average electric field  $10^4 \text{ V/m}$ . Nonetheless, for three pancakes the distribution of the current density (figure 6) is quite different, being more homogeneous in the

outermost pancake and more pronounced in the central one. Evidently, the electric field in the middle pancake is negligible because of the low magnetic field, yet, the electric field on the outermost pancakes achieves much larger values as compared with the one observed in the case of single pancake (figure 7).



### 3. Summary

A flexible algorithm and a Matlab program were developed for the calculation of current density, magnetic field and electric field distributions in HTS solenoids made of pancakes. This calculation takes into consideration the inhomogeneity of the current density distribution inside the anisotropic tape. The program can be easily adapted to any arbitrary shape of the coil and to an arbitrary distribution of the external magnetic field. The E-I curves of several single pancakes and coils were calculated and good conformity with measured E-I curves was observed. One can utilize the program in the coil design choosing his own criteria of coil's critical current, e.g., the average electric field  $10^4$  V/m over the coil, or the electric field  $10^4$  V/m at the weak point of the coil, or the energy dissipation in the entire coil, or distribution of local energy dissipation. The last criterion can be used for thermal runaway prediction provided a sufficient set of I-V curves in the relevant temperature interval exists.

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