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ScienceDirect
Physica C 460–462 (2007) 1247–1248
PHYSICA C
www.elsevier.com/locate/physc

Oscillating flux instability in vortex matter

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Available online 22 April 2007

Abstract

We explain theoretically an oscillatory behaviour in space and time of the magnetic induction in $Bi_2Sr_2CaCu_2O_{8+\delta}$ crystals during magnetic relaxation. This new "flux waves" phenomenon appears near the order–disorder vortex phase transition, under specific conditions of temperature and induction gradient. Our theory is based on two coupled equations: The Landau–Ginzburg equation for the order parameter of the vortex phase transition and the diffusion equation for the magnetic induction. Linear stability analysis of these equations shows an appearance of oscillatory instabilities characterized by a period and wavelength that are in accordance with the experimental results.

Keywords: High temperature superconductors; Vortex matter; Flux dynamics

1. Introduction

Oscillatory instabilities, both in space and time, were recently observed in the magnetic induction of Bi_2Sr_2Ca-$Cu_2O_{8+\delta}$ (BSCCO) crystals exposed to a steady magnetic field [1]. This "flux waves" phenomenon was observed in the vicinity of the vortex order–disorder phase transition line, in a region where the induction profile is relatively flat. The wave pattern exhibits a well defined wave length and amplitude, and it moves in the opposite direction to the incoming flux from the edge of the sample (vortices creep into the sample due to regular relaxation after application of an external magnetic field). In this paper we outline a theory which predicts spatiotemporal instabilities in the vortex matter under the conditions found experimentally, i.e. proximity to the order–disorder transition and a nearly flat profile.

2. Theory and discussion

In our theory we distinguish between the vortex phases by means of an order parameter Ψ : $\Psi = 0$ and $\Psi = \Psi_0(B)$ for the disordered and ordered vortex phases, respectively [2]. The evolution of $\Psi(x,\ t)$ is described by the Landau–Ginzburg dynamic equation:

$$\frac{1}{\Gamma}\frac{\partial\Psi}{\partial t} = -\frac{\delta F}{\delta\Psi}. \tag{1}$$

Here $F = \frac{1}{2}\int\left[D(\partial\Psi)^2 - \alpha\Psi^2 + \frac{1}{2}\gamma_0\Psi^4\right]$ is the free energy, and $\alpha = \alpha_0(1 - B/B_{od})$, D, γ_0, α_0 are the Landau expansion coefficients, Γ is the relaxation coefficient. The magnetic induction B is governed by a non-linear diffusion equation

$$\frac{\partial B}{\partial t} = \frac{c^2}{4\pi}\frac{\partial}{\partial x}\left[R_F\left(\frac{J}{J_c[\Psi]}\right)^{\sigma}\frac{\partial B}{\partial x}\right]. \tag{2}$$

Here, $J_c[\Psi]$ is the critical current density which changes smoothly from a large value at the disordered vortex phase to a small value at the ordered phase, and $\sigma = U_0/T$ where U_0 and T are the pinning potential and the temperature, respectively, and R_F is the flux flow resistance.

Eqs. (1) and (2) are coupled by the critical current $J_c[\Psi]$ which in our model is assumed to be

$$J_c[\Psi] \approx J_{c0}\exp(-n\Psi/\Psi_0), \tag{3}$$

where J_{c0} is the critical current in the disordered phase and n is a numerical model factor.

Eqs. (1)–(3) yield a time-independent solution: $B_0(x) \approx B_L + (B_R - B_L)x/L$, $\Psi_0(x) \approx \sqrt{\alpha_0(B_{od} - B_0(x))/\gamma_0 B_{od}}$,

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0921-4534/$ - see front matter
doi:10.1016/j.physc.2007.04.191

where B_L and B_R are the magnitudes of the magnetic induction at the left ($x = 0$) and right ($x = L$) sides of the sample, respectively.

Led by the experimental observations, we assume an almost flat induction profile. Presenting Ψ and B in the as $\Psi = \Psi_0(x) + \phi e^{iqx+\Omega t}$; $B = B_0(x) + b e^{iqx+\Omega t}$, where b and ϕ are the amplitudes of the perturbations one obtains after linearization of Eqs. (1)–(3)

$$\Omega(q)\begin{pmatrix} b \\ \phi \end{pmatrix} = \begin{pmatrix} iqF_3 - q^2F_2 & iqF_1 \\ -\psi_0 v_3 & -q^2 v_1 - \varepsilon \end{pmatrix}\begin{pmatrix} b \\ \phi \end{pmatrix},$$

$$v_3 = \frac{\alpha_0 \Gamma B_{od} B_{c2}}{4\pi R_n J_{c0}^2}; \quad v_1 = \frac{4\pi \Gamma D B_{c2}}{c^2 R_n B_{od}}; \quad \psi_0 = \sqrt{\frac{\alpha_0}{\gamma_0}};$$

$$F_1 = f_1 \frac{B}{B_{od}} \left|\frac{J_0}{J_{c0}}\right|^{\sigma} \frac{J_0}{J_{c0}}; \quad F_2 = f_0(\sigma + 1)\frac{B}{B_{od}}\left|\frac{J_0}{J_{c0}}\right|^{\sigma};$$

$$F_3 = f_0 \left|\frac{J_0}{J_{c0}}\right|^{\sigma} \frac{J_0}{J_{c0}}; \quad \varepsilon = 2\left(1 - \frac{B}{B_{od}}\right); \quad f_{0,} f_1 \sim 1.$$

A graphical description of the real and imaginary part of $\Omega(q)$ in units of $\Omega = 8\pi^2 R_n J_{c0}^2 / B_{c2} B_{od}$, $q = 8\pi^2 J_{c0}/cB_{od}$, for different parameters, is presented in Fig. 1 where we assume $\alpha_0 \Gamma = 10$ Hz [2] and $\varepsilon = 1 - B/B_{od} = 0.2$, for different slopes. The $\mathrm{Re}(\Omega)$ presents the well known Turing instability [3]. The range of q for which $\mathrm{Re}(\Omega) > 0$ and $\mathrm{Im}(\Omega) \neq 0$ presents an instability regime, characterized by oscillations in time and space with growing amplitude. The most unstable mode is characterized by a wave number q_m corresponding to the maximum in $\mathrm{Re}(\Omega)$ and by an oscillation frequency Ω_m corresponding to the $\mathrm{Im}(\Omega)$ at q_m.

Note that the instability described above falls into the category of threshold effects. This is demonstrated in Fig. 1. As one can see from this figure, the instability (i.e., positive $\mathrm{Re}(\Omega(q))$) appears above some threshold slope $J_0 \approx 0.0651 J_{c0}$.

In summary, we presented a theory predicting oscillatory behaviour of the induction in time and space in the vortex matter of superconductors near the vortex order–disorder phase transition. Our theory yields the size of the wavelength and period of the oscillations, as well as the temperature dependence of the period, which is in good accordance with the experimental data reported recently for BSCCO [1].

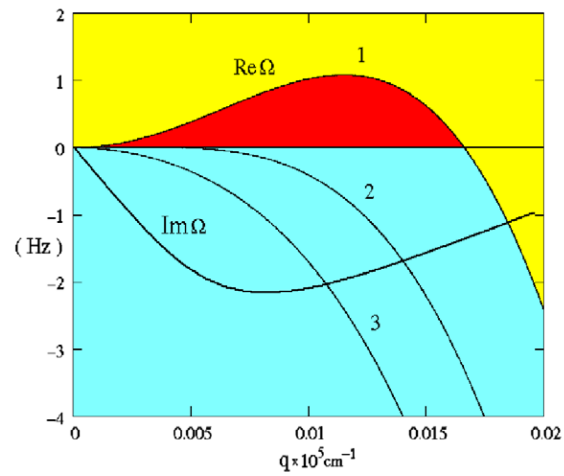

Fig. 1. Real and imaginary parts of $\Omega(q)$ for different induction slopes (in units of J_{c0}) $J_0 = 0.0171$ (1), 0.0165 (2), 0.0159 (3).

Acknowledgements

This research is supported by the Israel Science Foundation (ISF) – Centre of Excellence Program (Grant No. 8003/02) and the Heinrich Hertz Minerva Centre for High Temperature Superconductivity. Y.Y. and B.Ya.S. acknowledge support from the German–Israel Foundation (GIF). B.Ya.S. acknowledges a support from the ISF (Grant No. 4/03-11.7).

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